Large N behavior of Non-Ohmic Tunnel Junctions

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Non-Ohmic tunnel junctions are believed to occur in systems where the exciton and orthogonality catastrophe effects significantly modify tunneling rates. Here we present a simple non-perturbative treatment of the thermodynamic and transport properties of subOhmic and superOhmic tunnel junctions. Our analysis demonstrates the existence of a quantum phase transition in subOhmic but not superOhmic tunnel junctions. In addition, we find that the Coulomb gap vanishes continuously as the transition is approached.

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Recently there have been several papers which have emphasized the importance of the Fermi-edge singularities to electron tunneling phenomenon. These include a treatment of Coulomb blockade oscillations in double quantum dots by Matveev and Glazman [1] and the metal insulator transitions in granular materials by Drewes, Renn and Guinea [2]. In the later work, the authors argued that the metal-insulator transition observed by Herzog et al [3] in granular metallic wires was due to a quantum phase transition which occurs in sub-Ohmic tunnel junctions. Both these studies reflect the growing belief that exciton and orthogonality catastrophe effects are likely to be significant in a variety of systems including single tunnel junctions, quantum dot arrays, and granular metallic composite materials.

The interest in these effects first began with the classic treatments of x-ray absorption by Mahan Noziers and De Dominicus(MND) [4,5]. However, it was only a few years ago that the effect of Fermi edge singularities on tunneling phenomenon was first considered. Perhaps the first work examine these effects was by Ueda and Guinea [6,7] who showed that orthogonality catastrophe effects could cause the conductance to behave as $G \sim T^x$ where $x > 0$. A subsequent study of resonant impurity tunneling by Matveev and Larkin [8] predicted a zero temperature $I(V)$ characteristic of the form $I(V) \propto (V - V_{th})^{-\alpha}\theta(V - V_{th})$. This prediction was later confirmed [9] in an experiment involving resonant tunneling between a 2-dimensional electron gas (2DEG) and a localized impurity. Based on these works it is now generally believed that Fermi edge singularities a likely to produce a variety of interesting effects including non-Ohmic behavior in the $I(V)$ characteristics of tunnel junctions. For this reason, we will hereafter refer to such systems as non-Ohmic tunnel junction (NOTJ's).

In order to better understand the behavior of NOTJ's, we will present a simple yet powerful non-perturbative treatment based on the large N expansion. Our approach provides convenient alternative to a renomalization group treatment of NOTJ's. In particular, one can easily calculate critical exponents characterizing the metal Insulator transition which occurs in some subOhmic tunnel junctions. In addition, one can also obtain detailed expressions for conductances and specific heats. These calculations confirm the earlier prediction that sub-Ohmic tunnel junctions exhibit a quantum phase transition between an insulating state to a state with a divergent zero temperature conductance. In addition, the calculations indicate that the insulating state exhibits a renormalized Coulomb pseudo-gap which collapses in a continuous fashion as the transition is approached. Finally, the large-N approximation indicates that the metallic state is absent in Ohmic tunnel junctions.

In order to study the quantum dynamics of mesoscopic voltage fluctuations in NOTJ, will used a the long-range XY model introduced by Drewes et al. [2] This model is similar to that first introduced by Ben Jacob, Mottola, and Schon [10] to treat Ohmic tunnel junctions. The model is defined by the imaginary time partition function

$$Z = \int \mathcal{D}\mathbf{n}\ \delta(\mathbf{n}^2 - 1) \exp -S[\mathbf{n}] \tag{1}$$

where $\mathbf{n}(\tau) = (\cos\phi(\tau), \sin(\phi(\tau))$ may be used to relate the XY spin orientation to voltage using $\dot{\phi}(\tau) = eV/\hbar$. As discussed by Drewes et al [2], the effective action of NOTJ is

$$S[n] = \int d\tau d\tau' \alpha(\tau - \tau')[1 - \mathbf{n}(\tau) \cdot \mathbf{n}(\tau')] \tag{2}$$

where $\alpha(\tau) = \alpha_0 \tau_Q^{-\epsilon}/\tau^{2-\epsilon}$. The parameter α_0 is a dimensionless parameter proportional to the squared tunneling matrix element. For $\epsilon = 0$, this model reduces to that of Ben Jacob et al.

The above model has been analyzed Drewes et al using Monte Carlo simulations and the renormalization group. The renormalization group analysis shows that the model exhibits a correlation time

$$\xi_\tau \sim |\alpha_0 - \alpha_c|^{-1/\epsilon} \tag{3}$$

where the phase transition occurs at $\alpha_c = 1/2\pi^2\epsilon$. In the insulating phase one can associate $\hbar/\xi_\tau$ with a renormalized Coulomb gap subject to some important qualifications: (1.) The large α_0 (spin-wave) calculation demonstrate $dI/dV \sim T^{2(1-\epsilon)}$ behavior. So we expect that a

well defined Coulomb gap does not occur. However, at $T = 0$ the leading order perturbation theory does give a threshold at $E_Q = e^2/2C$. We expect therefore that higher order cotunneling processes will put states within the gap. So $\Delta \sim E_Q(\alpha_c - \alpha_0)^{-1/\epsilon}$ will not be a well defined gap although it may correspond to a pseudogap.

The existence of pseudogap together with its behavior near the phase transition should be observable in the behavior of the non-linear $I(V)$ characteristics. More generally, non-linear $I(V)$ characteristics could provide a powerful means to test the scaling properties of the transition. In particular, one expects such data according to the scaling ansatz scaling law [2]

$$dI/dV = \frac{e^2}{h}(k_BT/\Delta)^{\eta+1-\epsilon}F_{\pm}(eV/\Delta, k_BT/\Delta) \qquad (4)$$

where $F_-(x,y)$ and $F_+(x,Y)$ are the $\alpha > \alpha_c$ and $\alpha < \alpha_c$ branches of a universal scaling function which is finite for $x = y = 0$. The $V \to 0$ limit of this scaling form has been verified using Monte Carlo finite size scaling methods by Drewes et al [11] al.

A final consequence of the RG analysis is the relation $\eta = 1+\epsilon$ [12]. This result is very important since it implies a temperature independent conductance at the subOhmic to insulator critical point. Although the conductance is obtained from the fixed point Hamiltonian and is unaffected by irrelevant operators, it is not universal in any empirically useful sense. This unfortunate result follows from the fact that the model exhibits a line of fixed point Hamiltonians $\mathcal{H}_\epsilon$ which enable the "universal conductances" to take any value ranging from 0 to ∞. (See discussion below.)

At this point, we turn to our treatment of the large N approximation. We begin with a generalization of the long range XY model to a long range Heisenberg model. The dynamical degree of freedom of our model is now an N component unit-vector spin-field, $\hat{n} = (n_1, n_2, \ldots, n_N)$. Next we eliminate the $\hat{n}^2 = 1$ constraint by introducing an auxiliary field $\lambda(\tau)$ in eq. 1. This gives

$$\begin{aligned} Z = \int \mathcal{D}[\mathbf{n},\lambda]\ \exp - \int \tfrac{d\omega}{2\pi}\tfrac{\alpha_0 C_N}{2}|\omega|^{1-\epsilon}|\mathbf{n}(\omega)|^2 \\ - i\int d\tau\lambda(\tau)[\mathbf{n}^2(\tau)-1] \end{aligned}$$

where $C_N = -4\tau_Q^{-\epsilon}\Gamma(\epsilon-1)\sin(\epsilon\pi/2)$.

As $N \to \infty$, the functional integral is dominated by a saddle point on the imaginary $\lambda(\tau)$ axis [13]. Small fluctuations about the saddle point will correspond to a theory with a finite correlation length $\xi_\tau = \hbar/\Delta$. Hence, it will be convenient, to rewrite the auxiliary field in the form $\lambda(\tau) = \alpha_0 C_N[u(\tau) - i\Delta^{1-\epsilon}/2]$ Next we integrate out the $\mathbf{n}(\tau)$. This gives

$$\begin{aligned} S_{eff}(u) = \quad & \tfrac{E_Q}{k_BT}(\tfrac{N}{2}-1)\ln(C_N\alpha_0) - \\ & \int_0^{\hbar\beta} d\tau\alpha_0 C_N(\tfrac{\Delta^{1-\epsilon}}{2} + iu(\tau)) \\ & + \tfrac{N}{2}\ln[|-i\partial_\tau|^{1-\epsilon} + \Delta^{1-\epsilon} + 2iu(\tau)] \end{aligned}$$

We then obtain Δ by solving $\delta S_{eff}[u]/\delta u(\tau)|_{u=0} = 0$ or equivalently by solving

$$\alpha_0 C_N = N\int\frac{d\omega}{2\pi}\frac{1}{|\omega|^{1-\epsilon}+\Delta^{1-\epsilon}} \approx \frac{N}{\epsilon\pi}\left[E_Q^\epsilon - \Delta^\epsilon\right] \qquad (5)$$

This gives the

$$\Delta = E_Q(1-\frac{\alpha_0}{\alpha_c})^{1/\epsilon} \qquad (6)$$

where

$$\alpha_c = -\frac{N}{4\pi\epsilon}\frac{1}{\Gamma(\epsilon-1)\sin(\frac{\epsilon\pi}{2})} \qquad (7)$$

The $\epsilon \to 0$ limit, behavior of the Δ is

$$\Delta = E_Q\exp - \frac{2\pi^2\alpha_0}{N} \qquad (8)$$

Next we consider spin-fluctuations about the saddle point. In the large N limit, $G(\tau-\tau') = <\mathbf{n}(\tau)\cdot\mathbf{n}(\tau')>$ is given by

$$G(\omega) = \frac{N}{\alpha_0 C_N}\frac{1}{|\omega|^{1-\epsilon}+\Delta^{1-\epsilon}} \qquad (9)$$

This result demonstrates that $\xi_\tau = \hbar/\Delta$ is indeed a correlation length. The fact that ξ_τ diverges when $\alpha_0 = \alpha_c$ then implies that $\alpha_c = N/2\pi^2\epsilon$ is a critical point. This leading order in $1/N$ result is, of course, consistent with the renormalization group results that $\alpha_c = (N-1)/2\pi^2\epsilon[1+O(\epsilon)]$. At the critical point the spin-spin correlation function becomes

$$G(\tau) = \Gamma(\epsilon+1)\cos(\frac{\epsilon\pi}{2})\left(\frac{\tau_Q}{\tau}\right)^\epsilon \qquad (10)$$

This form implies $\eta = 1+\epsilon$ as was previously obtained by Fisher Ma and Nickle [12] and is believed to be exact.

Next we wish to consider the behavior of the dimensionless XY model specific heat. The quantity under discussion is $C_{XY} \equiv \alpha_0^2 d^2\ln Z/d\alpha_0^2$. This is not the physical specific heat of a tunnel junction. It is, however, a quantity which is relevant to Quantum Monte Carlo studies [14] of the tunnel junction. The calculation the heat capacity proceeds as follows. First, we calculate $F \equiv -k_BT\ln Z$ to leading order in $1/N$ using the $u(\tau) = 0$ saddle point. This gives a zero temperature free energy

$$\begin{aligned} F = \tfrac{N}{2}E_Q\ln(C_N\alpha_0) - \tfrac{1}{2}\alpha_0 C_N\Delta^{1-\epsilon} \\ + \tfrac{N}{2}\int\tfrac{d\omega}{2\pi}\ln\left(|\omega|^{1-\epsilon}+\Delta^{1-\epsilon}\right) \end{aligned}$$

This coincides with $k_BTS[u=0]$. Now using the Euler-Lagrange equation for Δ together with eq. 6 one obtains the expression

$$\frac{C_{XY}}{N\beta E_Q} = \frac{1}{2} - \frac{1}{2\pi}\frac{1-\epsilon}{\epsilon^2}\left(\frac{\alpha_0}{\alpha_c}\right)^2\left(1-\frac{\alpha_0}{\alpha_c}\right)^{\frac{1}{\epsilon}-2} \qquad (11)$$

This result is valid to leading order in $O(1/N)$ provided that $\alpha_0 < \alpha_c$ and $|\alpha_c - \alpha_0|/\alpha_0$ are both small. One sees that the specific heat exponent $\alpha = 2 - 1/\epsilon$ coincides with that obtained from the Josephson relation $\alpha = 2 - d\nu$.

At this point, we wish to consider the behavior of the dc conductance. First we generalize the electric current. For the physical $U(1) = O(2)$ model, the current is obtained using $I(\tau) = -\delta S[A_x]/\delta A_x(\tau)$ where

$$\frac{S[A_x]}{\hbar} = \int d\tau d\tau' \alpha(\tau - \tau')[1 - \cos(\Phi(\tau) - \Phi(\tau'))] \quad (12)$$

where $\Phi(\tau) \equiv \phi(\tau) - \frac{e}{\hbar} A_x(\tau)$. This gives

$$I(\tau) = 2e \int d\tau' \alpha(\tau - \tau') \sin(\phi(\tau) - \phi(\tau')) \quad (13)$$

Equivalently, an expression for the electric current could be obtained by considering the variation of S under a local gauge transformation of the form $\delta\phi(\tau) = \delta\epsilon(\tau)$. Then $\frac{e}{\hbar}\delta S/\delta\epsilon(\tau) = I(\tau)$. This second definition is convenient for defining the electric current(s) in the $O(N)$ model. In the O(N) model one has a set of $N-1$ electric currents, $I^a(\tau)$, $a = 1 \ldots N-1$ each associated with the $N-1$ Lie algebra generators $\mathbf{T}^a$. Since $\mathbf{n}(\tau)$ is an N component vector field which transform according to the fundamental representation of $O(N)$, the $\mathbf{T}^a$, matrices are $N \times N$ anti-symmetric matrices which can be chosen such that $\mathrm{Tr}\mathbf{T}^a\mathbf{T}^b = \delta_{ab}$. We now define the electric currents as follows: Under a local gauge transformation $\delta\mathbf{n}(\tau)/\delta\epsilon^a(\tau') = \mathbf{T}^a \cdot \mathbf{n}(\tau)\delta(\tau - \tau')$. The currents are then defined to be $I^a(\tau) = \frac{e}{\hbar}\delta S[\epsilon]/\delta\epsilon^a(\tau)$. From this we obtain the explicit form of the electric currents

$$I^a(\tau) = 2e \int d\tau' \alpha(\tau - \tau')\vec{n}(\tau) \cdot \mathbf{T}^a \cdot \vec{n}(\tau') \quad (14)$$

The special case of the $O(2) = U(1)$ model, the above result reduces to eq. 13 as required. This may be shown by taking $\mathbf{T} = i\sigma_y$ and $\mathbf{n}(\tau) = (\cos(\phi(\tau)), \sin(\phi(\tau')))$.

Having defined the $O(N)$ electric currents, we want to perform a Kubo calculation of the dc conductances G^{ab}. The first step is to use the identity

$$< \frac{\delta S}{\delta\epsilon^a(\tau)} \frac{\delta S}{\delta\epsilon^b(\tau')} >= - < \frac{\delta^2 S}{\delta\epsilon^a(\tau)\delta\epsilon^b(\tau')} > \quad (15)$$

which follows from a functional integration by parts. This gives a current current correlation function for the physical model (i.e. $U(1) = O(2)$ model) of the form [15]

$$< I^a(\tau) I^b(\tau') >= -\frac{2e^2}{\hbar}\alpha(\tau - \tau') < \mathbf{n}(\tau) \cdot \mathbf{T}^a\mathbf{T}^b \cdot \mathbf{n}(\tau') > \quad (16)$$

Using the Kubo formula and eqs. 10 and 16 we obtain the critical conductance, G_c, of the tunnel junction to leading order in $1/N$:

$$G^{ab} = 2\pi(1 - \epsilon)\mathrm{ctn}\left(\frac{\epsilon\pi}{2}\right)\frac{e^2}{h}\delta_{ab} = G_c\delta_{ab} \quad (17)$$

Because of the value of the η exponent the conductance is temperature independent. Next we observe that the conductance diverges as $\epsilon \to 0$. See fig. 1. This means that for small ϵ a rather large value of α_0 (or equivalently the tunneling matrix element) is required in order to obtain a subohmic phase. This is consistent with the belief that the ordered (subOhmic) phase is destroyed as $\epsilon \to 0$ and that the phase transition is absent for positive ϵ. The second observation is that as $\epsilon \to 1$ $\sigma \to 0$. This is consistent with the fact that the disordered (insulating) phase is absent for $\epsilon \geq 1$. In particular, it implies that an arbitrarily weak α_0 will order the XY model as $\epsilon \to 1$ is approached. The fact that the critical conductance can, depending on the value of ϵ, range from 0 to ∞ is interesting since it implies that no universal value of the conductance should be expected.

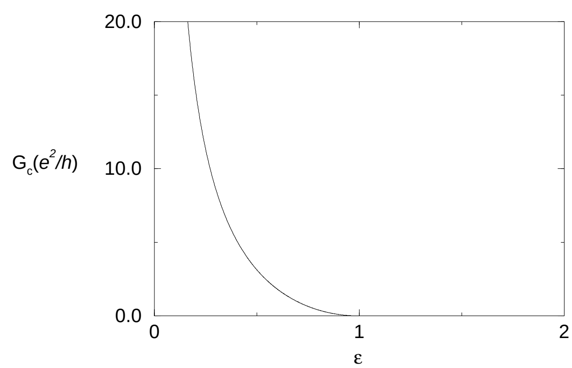

FIG. 1. The critical conductance vs. ϵ. Observe that the insulating phase is squeezed out of the phase diagram (i.e. $G_c \to 0$) as $\epsilon \to 1$. Similarly the conducting phase is squeezed out as $\epsilon \to 0$.

Next we consider the AC conductance $G(\omega)$. For this purpose it is convenient to approximate $< \mathbf{n}(\tau) \cdot \mathbf{n}(0) >$ as being given by its critical behavior for $\tau << 1/\Delta$ and exhibiting a cutoff for $\tau \sim 1/\Delta$ i.e. we assume that

$$G(\tau) \approx \Gamma(1 + \epsilon)\cos(\frac{\epsilon\pi}{2})\frac{\tau_Q^\epsilon}{\tau^\epsilon}\exp -(\Delta|\tau|) \quad (18)$$

With this approximate form one can then obtain the time-ordered real time current current correlation function:

$$C_t^{ab}(\omega) \equiv -i\int_{-\infty}^{\infty} dt \ < T_t(I^a(\tau)I^b(0)) > \exp i\omega t \quad (19)$$

Using eq. 16 one obtains

$$C_t^{ab}(\omega) = -2\Delta G_c \delta_{ab} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{|\omega'|}{(\omega'-\omega)^2 - \Delta^2 + \imath\delta} \quad (20)$$

This in conjunction with the the Kubo formula is then used to obtain the the ac conductance $G(\omega)^{ab} = [G'(\omega) + iG''(\omega)]\delta_{ab}$. The real part is found to be

$$G'(\omega) = G_c(1 - \frac{\Delta}{2|\omega|})\theta(|\omega| - \Delta) \quad (21)$$

where $\theta(x) = 0$ for $x < 0$ and 1 for $x > 0$ and where G_c is the critical conductance as given by eq. 17. This result allows one to identify Δ as an excitation gap which, near the transition, is much smaller than the charging energy E_Q.

Previously, Drewes et al [2] found a well defined Coulomb gap in a leading order ($O(\alpha_0)$) perturbative calculation of the $I(V)$ characteristic. However a α_0^2 calculation of $G(T)$ found that

$$G \sim \frac{\alpha_0^2}{R_Q} \left(\frac{\pi k_B T}{E_Q} \right)^{2(1-\epsilon)} \quad (22)$$

That result indicates that the conductance does not exhibit an activated temperature dependence and that higher order processes introduce states inside the gap. Moreover, one would argue that the zero temperature $I(V)$ characteristic is of the form $I(V) \sim (e^2/h)\alpha_0^2(eV/\Delta)^{2(1-\epsilon)}$ for $V << \Delta$ and that ac conductance is of the form

$$G'(\omega) \sim (\omega/\Delta)^{2(1-\epsilon)} \quad (23)$$

when $\omega << \Delta$. These results presumable occur at higher than leading order in the $1/N$ expansion.

In addition to calculating the $G'(\omega)$ one can also calculate the imaginary part of the conductance. In this case one finds

$$G''(\omega) = \frac{C_t(0)}{\omega} + \frac{\Delta}{\pi\omega} G_c \left[\ln \left| \frac{\Delta^2}{\Delta^2 - \omega^2} \right| + \frac{\omega}{\Delta} \ln \left| \frac{\omega - \Delta}{\omega + \Delta} \right| \right] \quad (24)$$

where $C_t(0)$ depends logarithmically on E_Q/Δ.

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